1. Accidents at an intersection depend on road conditions: Dry (D), Wet (W), or Icy (I). The probabilities of an accident (A) are 0.001 if dry, 0.10 if wet and 0.30 if icy. It is dry 80 % of the time, wet 15 %. Dry, wet, and icy are mutually exclusive and collectively exhaustive.

(a) What is the probability of an accident?

$P(A) = P(A|D)\ P(D) + P(A|W)\ P(W) + P(A|I)\ P(I)$

$P(A) = 0.001 \times 0.8 + 0.1 \times 0.15 + 0.3 \times 0.05 = 0.0008 + 0.015 + 0.015 = 0.0308 = \underline{0.03}$

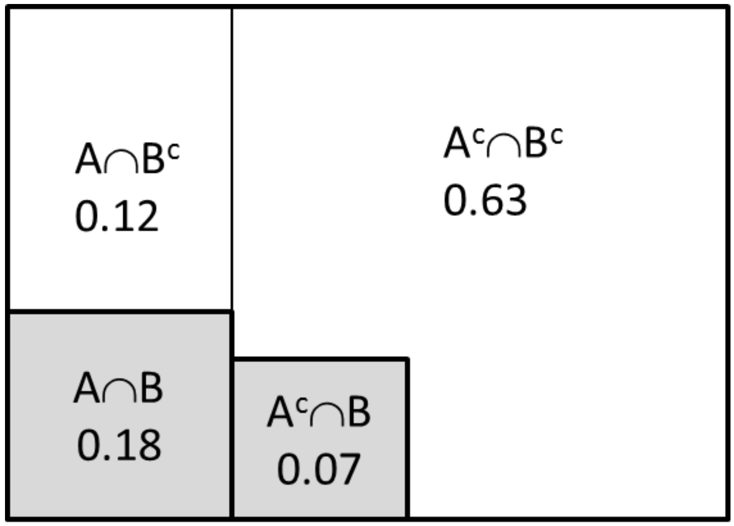
(b) What is the probability that three such intersections have no accident? Assume the accident probability is independent.

$P(A_1^c \cap A_2^c \cap A_3^c) = P(A_1^c)\ P(A_2^c)\ P(A_3^c) = (1 - 0.0308)^3 = 0.9104 = \underline{0.91}$

2. Three intersections are arrayed along a straight road. The probability of a red light at each intersection is A, B, and C, respectively. Given: $P(A) = 0.3$, $P(B|A) = 0.6$, $P(B|A^c) = 0.1$, $P(C|B) = 0.6$, $P(C|B^c) = 0.1$, and C is independent of A.

(a) P(B) = ?

$P(B) = P(B|A)\ P(A) + P(B|A^c)\ P(A^c) = 0.6 \times 0.3 + 0.1 \times 0.7 = 0.18 + 0.07 = \underline{0.25}$ (ToTP)



(b) What is the probability that all three are green?

$P(C^c \cap A^c \cap B^c) = P[(C^c \cap (A^c \cap B^c)] = P[C^c | (A^c \cap B^c)]\ P(A^c \cap B^c)$

Because C and A are independent: $P[C^c | (A^c \cap B^c)] = P(C^c | B^c)$

And, using the complement: $P(C^c | B^c) = 1 - P(C | B^c)$

So, $P(C^c \cap A^c \cap B^c) = [1 - P(C|B^c)]\ P(A^c \cap B^c) = (1 – 0.1)\ 0.63 =$ $\underline{0.567}$

(c) $P(C) = ?$

$P(C) = P(C|B)\ P(B) + P(C|B^c)\ P(B^c) = 0.6 \text{x} 0.25 + 0.1 \text{x} 0.75 = 0.15 + 0.075 =$ $\underline{0.225}$
(ToTP)

(Note: $P(C|B)\ P(B) = P(C \cap B) = 0.15$ and $P(C \cap B^C) = 0.075$)